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Virtual SEA: towards an industrial process

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ABSTRACT

In the high frequency range, the SEA method has been applied to air borne path with success to predict both internal and external sound environment. Nevertheless, structure-borne prediction is still at issue -especially for cars, in the range 200 to 2000 Hz- as results are widely dependant on subsystem partition and validity of various assumptions required by SEA.
Experimental SEA test technique (ESEA), applied to car bodies, has brought to the fore that SEA power balanced equations could robustly describe structure-borne noise. To make ESEA predictive, the database of measured FRF is simply replaced and enlarged by synthesized data generated from a finite element (FE) model and a selected observation grid of nodes. This technique, called Virtual SEA (VSEA), has been presented at SAE/NVC 2003. Since then, many developments have been carried out to improve the general efficiency of the three main steps of VSEA:

- The generation of the band-integrated FRF database (between nodes of the observation grid) is now integrated in the virtual SEA solver. As several millions of band averaged FRF can be required for a full car body analysis, a specific fast FRF synthesis is performed using analytical integration and band-optimized solutions by limiting the number of cross-modal terms in the series solution.
- Automatic subsystem partition has been widely improved by developing different algorithms that limit the sensitivity of results to initial conditions.
- ESEA loss matrix identification process has been reviewed for better resolution of highly non homogeneous system such as car bodies. The normalization of squared FRF matrix by the input mobility matrix allows a direct determination of subsystems modal density. The dynamics of a very large 3D FE model can thus be compressed into a small SEA loss matrix while preserving local transfer information at all nodal observation points.

Examples of applications are shown on car components.

INTRODUCTION

Applications of SEA to car sound transmission have demonstrated that SEA method can predict air-borne transmission path with good success. The structure-borne sound prediction was found to be more challenging -mainly in the range 200 to 2000 Hz- as results are widely dependant on subsystem partition and validity of various “fuzzy” assumptions required by SEA. By comparing results from both "Analytical" and "Experimental" SEA (ESEA) modeling, it was shown that ESEA models could robustly describe structure-borne noise [1,2], meaning a SEA representation of energy transfers in a car body was achievable while analytical models were found very sensitive to user's choice due to a priori assumptions in subsystem decomposition and lack of theory to predict indirect structural coupling loss factors (CLF).

To make ESEA predictive, the database of measured FRF was simply replaced and enlarged by synthesized data generated from a Finite Elements (FE) model. This technique, called Virtual SEA, was presented at the SAE conference NVC 2003 [3] and was showing very promising results. Predicted transfers in a car chassis ware within 2 dB from measured data while classical SEA analytical model was away from 10 to 20 dB.

Since then, a tentative work was carried out to give some theoretical background to the technique. At the same time, relying on the new theoretical developments, the computational efficiency was improved, especially for transfer functions computations and for the automatic sub-structuring procedure.

In the first part of the paper, the path from the FEM results to a reduced non-dimensional power balance will be explained. Then, the identification of a SEA model to the reduced model will be shown. At least, computational improvements will be explained. But first, let us define the modeling framework.

MODELING FRAMEWORK

The framework of the present development is the linear theory of structural vibrations of finite-sized systems. The considered systems are modeled using their linear constitutive equations and boundary conditions [4]. The resulting problem is solved with a Finite Element Method [5] either using local coordinates or generalized coordinates (modes). In the frequency domain, the problem takes the form:

$$\left[\mathbf{K} + j\mathbf{D} - \omega^2\mathbf{M}\right] \cdot \mathbf{x} = \mathbf{f} \quad (1)$$

where $\mathbf{f}$ is the complex vector of external forces, $\mathbf{x}$ is the complex displacement, $\mathbf{K}$ and $\mathbf{M}$ are respectively the stiffness and mass matrices, and $\mathbf{D}$ is a damping matrix. All matrices have good positivity properties.

Equation (1) may also be written

$$\mathbf{A}\mathbf{x} = \mathbf{f}, \quad (2)$$

with $\mathbf{A}(\omega) = \left[\mathbf{K} + j\mathbf{D} - \omega^2\mathbf{M}\right]$ is introduced as the dynamic stiffness matrix.

The inverse expression provides the displacements of the structure, for given external loads. For energy approaches, it is convenient to introduce the velocity of the structure, $\mathbf{v} = j\omega\mathbf{x}$, so that :

$$\mathbf{v} = \mathbf{T} \cdot \mathbf{f} \quad (3)$$

where $\mathbf{T}(\omega) = j\omega\mathbf{A}(\omega)^{-1}$ is called the mobility transfer matrix of the structure.

The mobility transfer matrix is a square complex symmetric matrix. As car body structures are mainly made of shells, only normal degrees of freedom can be considered, assuming no in-plane –or tangent- resonance occurs in the studied frequency range. More precisely underlying in-plane energy currently shared by shells in the low frequency is always taken into account in the transfers as it contributes to normal velocity. As an option, it is also possible to capture the mobility transfer matrix in the direction of maximum square velocity in both input and output directions of the local stress-to-velocity tensor, avoiding normal direction assumptions, useful for complex 3D geometry. Then if in-plane energy is becoming predominant above a given frequency, the model will converge to in-plane behavior.

Due to computing limitations, the observation of the structure is limited to a reduced number of observation points, assuming its statistical dynamic behavior can be captured from a coarse sampling.

In the following, we will introduce the real part of the input mobility matrix as a vector:

$$\mathbf{y} = diag\left\{\mathrm{Re}\left\{\mathbf{T}\right\}\right\} \quad (4)$$

The diagonal input mobility matrix will be noted $\mathbf{Y}$, such as:

$$\mathbf{Y} = \mathbf{diag}\left\{\mathbf{y}\right\} \quad (5)$$

Where $\mathbf{diag}\left\{\mathbf{y}\right\}$ is a diagonal matrix, which diagonal values are equal to the vector $\mathbf{y}$, with the property $\mathbf{y} = diag\left\{\mathbf{diag}\left\{\mathbf{y}\right\}\right\}$.

TOWARD A MID-HIGH FREQUENCIES DATA STATISTICAL REDUCTION

RANDOM EXCITATION

Most of the structural modeling work wishes to get a model that applies for any possible load case. One reason is that, during the vibroacoustic design process, one of the first questions to answer is “where to connect the vibration sources?” or “how to design connection points”. As far as the sources position and their intensities are still unknown, a stochastic modeling has to be used.

Classically, the “rain-on-the roof” [6] assumption is used; applied forces are modeled as uncorrelated point forces with equal intensity in average. If one notes $\tilde{\mathbf{f}}$ a vector of random forces, following the “rain-on-the-roof” definition, one may write the mathematical likelihood of the external forces cross-spectral matrix as:

$$E\left\{\tilde{\mathbf{f}}\tilde{\mathbf{f}}^{\mathbf{t}}\right\} = \mathbf{diag}\left\{\mathbf{f}^{\mathbf{2}}\right\} \quad (6)$$

where $\mathbf{f^2}$ is the vector of the power spectral density of the uncorrelated forces applied to each point of the structure.

In this paper, we will consider a slightly different expression because we work on structures that may be strongly heterogeneous because they are designed on purpose. When looking to the structure vibrations from an SEA point of view, vibration levels are related to the power inputs and not applied forces. This implies that the most robust design regarding interior noise is obtained when all sources in the same subsystem provide the same power. Practically, assuming we deal with force sources [7], this leads to a design of sources connection points where input mobilities are such as the differences between injected powers are minima.

Introducing the cross-spectral power matrix between applied forces and structure velocities, expression (6) is replaced by:

$$E\left\{\mathbf{Y}\tilde{\mathbf{f}}\tilde{\mathbf{f}}^{\mathbf{t}}\right\} = \mathbf{diag}\left\{\boldsymbol{\pi}\right\} \quad (7)$$

where $\boldsymbol{\pi}$ is the mean power input into the structure such as,

$$\boldsymbol{\pi} = \mathbf{Y} \cdot \mathbf{f}^2 \tag{8}$$

It may be noticed here that the proposed stochastic modeling of excitations tends toward the classical "rain-on-the-roof" when the considered structure is homogeneous.

RANDOM RESPONSE OF THE STRUCTURE

The random response of the structure, $\tilde{\mathbf{v}}$, to the random applied forces, $\tilde{\mathbf{f}}$, is defined by the likelihood of a cross-spectral matrix

$$\tilde{\mathbf{v}}\tilde{\mathbf{v}}^{\mathbf{t}} = \mathbf{T}\tilde{\mathbf{f}}\tilde{\mathbf{f}}^{\mathbf{t}}\mathbf{T}^{\mathbf{t}} \tag{9}$$

As explained above, we are interested in point-square velocities and wish to compute the likelihood, *E*, of the cross-power spectral density vector:

$$\mathbf{v}^2 = E\left\{ diag\left\{ \tilde{\mathbf{v}}\tilde{\mathbf{v}}^{\mathbf{t}} \right\} \right\} \tag{10}$$

Equations (8) to (10) leads to

$$\mathbf{v}^2 = diag\left\{ \mathbf{T}\mathbf{Y}^{-1}\mathbf{diag}\left\{ \boldsymbol{\pi} \right\}\mathbf{T}^{\mathbf{t}} \right\} \tag{11}$$

It can be demonstrated that expression (11) can be favorably put into the form:

$$\mathbf{v}^2 = \mathbf{T}^2\mathbf{Y}^{-1} \cdot \boldsymbol{\pi} \tag{12}$$

where $\mathbf{T}^2$ is the real matrix of the term-to-term squared value of $\mathbf{T}$.

It is noticed here that $\mathbf{v}^2$ is a quantity that can be measured, and therefore, computations will always have to come back to $\mathbf{v}^2$.

NON-DIMENSIONAL POWER TRANSFER BALANCE

The authors propose here to introduce the, so-called, local modal energy:

$$\mathbf{e_n} = \frac{1}{4}\mathbf{Y}^{-1}\mathbf{v}^2 \tag{13}$$

Using definition (13), expression (12) is equivalent to:

$$\mathbf{e_n} = \frac{1}{4}\mathbf{Y}^{-1}\mathbf{T}^2\mathbf{Y}^{-1}\boldsymbol{\pi} \tag{14}$$

This last expression is a non-dimensional, symmetric relationship which is a good preset for further mathematical deductions as also shown in [15].

FREQUENCY AVERAGING

Medium and high frequencies are frequency domains where it is accepted that the dynamic response of a structure is dominated by stochastic effects, so that only average quantities have enough robustness to be worked out.

Medium frequencies differ from high frequencies in the fact that some of the dynamical components still behave as low frequency; that is to say they are influenced by their boundary conditions and their geometry, or, in other words, controlled by few vibration modes. High frequencies, to the contrary, means a high enough number of modes involved in the responses so that asymptotic approaches apply.

The band-integrated modal energy, over a frequency band B, is defined as:

$$\mathbf{e_n}\big|_B = \int_B \mathbf{e_n}(f)df \tag{15}$$

Substituting the local modal energy expression (14) in (15), and assuming a constant power spectral density π_0 for the injected power vector over B,

$$\boldsymbol{\pi}\big|_B = |B|\boldsymbol{\pi}_0 \tag{16}$$

It comes:

$$\mathbf{e_n}\big|_B = \mathbf{E}_{\mathbf{n}B}\,\pi\big|_B \tag{17}$$

where the symmetric Modal Transfer Energy Matrix, $\mathbf{E}_{\mathbf{n}B}$, is defined as

$$\mathbf{E}_{\mathbf{n}B} = \frac{1}{4|B|}\int_B \mathbf{Y}^{-1}\mathbf{T}^2\mathbf{Y}^{-1}df \tag{18}$$

and corresponds to the mean modal energy response of the system over the band B to unit-injected power.

At the moment, the modal transfer energy matrix is computed in an approximated way as:

$$\mathbf{E}_{\mathbf{n}B} = \frac{1}{4}\left\langle \mathbf{Y} \right\rangle_B^{-1}\left\langle \mathbf{T}^2 \right\rangle_B\left\langle \mathbf{Y} \right\rangle_B^{-1} \tag{19}$$

For practical applications, 1/3 octave band are used.

It is quotable, as shown on figure 1, that this last matrix has a better definite numerical structure than the previously considered, $\left\langle \mathbf{T}^2 \right\rangle_B$. The dynamic range of point-to-point transfers is reduced from 94 down to 39 dB by mobility vector normalization.

This is a key-point when trying to apply an automatic partitioning process into subsystems based on the optimization of the numerical organization of matrices as well as measuring the errors attached to the statistical modeling.

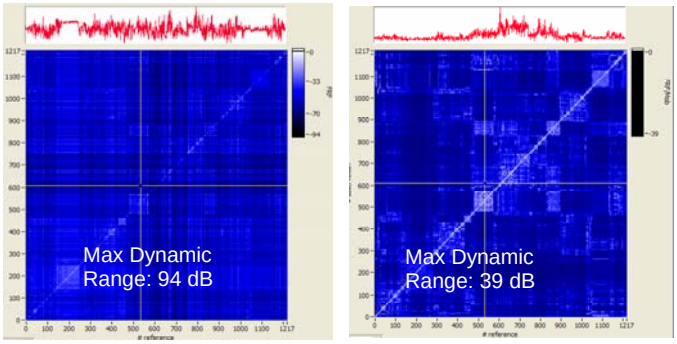


Figure 1: On the left transfer mobility matrix of a car body $\left\langle \mathbf{T}^2 \right\rangle_B$ averaged over 630 Hz 1/3 octave band; On the right, transfer modal energy matrix $\mathbf{E}_{\mathbf{n}B}$

SPACE AVERAGING AND SUB-STRUCTURING

The main goal of this work is to provide a reduced model of the structure dynamical behavior that is robust to local heterogeneity -thanks to the local modal energy definition-. The likelihood of the local modal energy is then assumed to be equal to the space-average over a given space domain, e.g. a subsystem. The space-averaged modal density over a given subsystem, Ω_i, is defined as :

$$\left\langle \mathbf{e_n} \right\rangle_{B,\Omega_i} = \frac{1}{S_{\Omega_i}} \int_{\Omega_i} \mathbf{e_n}\big|_B \, dM_{\Omega_i} \tag{20}$$

where $S_{\Omega_i} = \int_{\Omega_i} dM_{\Omega_i}$ measures the extend of the considered structural subsystem.

Practically, the space-averaged modal energy is computed as the discrete mean of local sampled values:

$$\overline{e_{n,B\Omega_i}} = \frac{1}{N_i} \sum_{k_i=1..N_i} e_n\big|_{B,k_i} \tag{21}$$

where N_i is the number of observation points belonging to Ω_i, and k_i is the local point index.

Introducing the rectangular sub-structuring matrix, $\mathbf{S}$, such as,

$$S_{\Omega i,k} = \begin{vmatrix} 0 & \text{when } \mathrm{M_k} \notin \Omega_i \\ \dfrac{1}{N_i} & \text{when } \mathrm{M_k} \in \Omega_i \end{vmatrix} \tag{22}$$

the vector of averaged modal energies can be expressed as:

$$\overline{\mathbf{e}_{\mathbf{n},B\Omega}} = \mathbf{S}_\Omega \, \mathbf{e_n}\big|_B \tag{23}$$

In the same manner, the vector of the averaged input power within each subsystem is written as:

$$\overline{\boldsymbol{\pi}_{B,\Omega}} = \mathbf{S}_\Omega^{++} \, \boldsymbol{\pi}\big|_B \tag{24}$$

Where $\mathbf{S}_\Omega^{++} = \left(\mathbf{S}_\Omega^{\mathbf{t}} \mathbf{S}_\Omega\right)^{-1} \mathbf{S}_\Omega$ is the pseudo-inverse of $\mathbf{S}_\Omega$

It appears that the pseudo-inverse may be computed easily, and it is defined by:

$$S^{++}_{\Omega_i,k} = \begin{vmatrix} 0 & \text{when } \mathrm{M_k} \notin \Omega_i \\ 1 & \text{when } \mathrm{M_k} \in \Omega_i \end{vmatrix} \tag{25}$$

The average modal energy transfer between subsystems is then deduced from equations (17), (23), (24)

$$\overline{\mathbf{e}_{\mathbf{n},B\Omega}} = \mathbf{E}_{\mathbf{n},B\Omega} \, \overline{\boldsymbol{\pi}_{B,\Omega}} \tag{26}$$

where $\mathbf{E}_{\mathbf{n},B\Omega} = \mathbf{S}_\Omega \mathbf{E}_{\mathbf{n},B} \mathbf{S}_\Omega{}^{\mathbf{t}}$ is the reduced modal energy transfer matrix of the studied system.

Equation (26) provides the likelihood estimate of the frequency band-averaged local modal energies that is derived from the fundamental equations of the problem, solved using a finite elements method. Therefore, Equation (26) constitutes a reduced statistical model. The only assumption made until now is sources independence and the accuracy of results is related to the sampling of observation points. Of course, equation (26) is strongly related to the given subsystem partition that is considered. In the following sections, it will be shown how this model can be identified to an SEA model and how the automatic sub-structuring is performed in an optimal way regarding SEA.

This is basically what Virtual SEA means for the authors: identifying a SEA model to a statistical reduction of FE results on selected sub-structures (or subsystems). Its main advantage is to provide a fully consistent modeling basis for any complex structure that can be modeled using a FE method.

ESEA PRINCIPLES

Before introducing the Virtual SEA formulation, it is needed to come back to the experimental SEA, that basically remains the framework of Virtual SEA. ESEA is a technique that has been developed from more than 20 years [1, 8, 9], mainly to build and validate hybrid SEA models that include both measured and/or predicted (analytical) SEA parameters. It mainly consists in the identification of the SEA model associated to given sub-structured system, from measured Frequency Response Functions (FRF).

In a direct SEA approach of a problem, the injected power, loss factors, modal density and masses are known parameters, obtained from analytical formulas and subsystem energies are the unknowns. SEA parameters are linked by the power-balanced set of equations, describing the energy conservation of the system, integrated over a frequency band B:

$$\boldsymbol{\pi}_{\Omega}\big|_B = \omega_B \mathbf{L}_{B,\Omega} \cdot \mathbf{e}_{\Omega}\big|_B \tag{27}$$

where $\boldsymbol{\pi}_{\Omega}\big|_B$ is the vector of powers applied by external forces in each subsystem, $\mathbf{e}_{\Omega}\big|_B$ is the vector of total vibrational energy stored within subsystems, $\mathbf{L}_{B,\Omega}$ the loss matrix in-between the structured system $\Omega = \cup \Omega_i$, in the frequency band B.

From here, considering all variables are averaged over space and integrated over frequency, the indices B and Ω will be omitted. Moreover, in order to remind that energies are obtained from a mean over a number of discrete observation points an upper bar is used for estimated average values. Hence, equation (27) is re-written in a more suitable form:

$$\bar{\boldsymbol{\pi}} = \omega_B \mathbf{L} \cdot \bar{\mathbf{e}} \tag{28}$$

Assuming the system is resonant enough so that the potential and kinetic energies are equal, the subsystems energy vector is generally approximated as:

$$\bar{\mathbf{e}} = \mathbf{diag}\{\mathbf{m}\} \cdot \overline{\mathbf{v}^2} \tag{29}$$

where $\mathbf{m}$ is the vector of subsystems equivalent masses for the frequency band B.

The loss matrix is build from loss factors (damping loss factor , η_i ; coupling loss factor, η_{ij}) as follows:

$$L_{ij} = \begin{cases} \eta_i + \sum_{i \neq j} \eta_{ij} \text{ when } i = j \\ -\eta_{ji} \text{ when } i \neq j \end{cases} \tag{30}$$

Coupling loss factors are assumed to satisfy the well-known SEA reciprocity relationship:

$$n_i \eta_{ij} = n_j \eta_{ji} \tag{31}$$

when introducing the modal density of subsystem i, n_i .

This relationship denotes the necessity of the existence of a symmetrical formulation inherent to the linear theory of vibrations. Thus, the symmetrical modal loss matrix may be introduced:

$$\mathbf{L}_{\mathbf{n}} = \mathbf{L} \cdot \mathbf{diag}\{\mathbf{n}\} \tag{32}$$

where $\mathbf{n}$ is the vector of the number of modes in the subsystems.

Using (32), equation (29) can be written in a symmetrical form:

$$\bar{\boldsymbol{\pi}} = \omega_B \mathbf{L}_{\mathbf{n}} \cdot \overline{\mathbf{e}_{\mathbf{n}}} \tag{33}$$

where $\overline{\mathbf{e}_{\mathbf{n}}} = \mathbf{diag}\{\mathbf{n}\}^{-1} \cdot \bar{\mathbf{e}}$ is the vector of modal energies and ω_B is the angular frequency at the central frequency of band B

This last relationship appears to be the inverse form of equation (26) which can be computed using a finite element modeling. This is where SEA and Virtual-SEA merge.

In ESEA, the loss matrix is the unknown while the averaged (space and band average) power input and the averaged subsystem velocities are measured. The first step of the method consists in estimating the subsystems energy. In a second step the loss factors can be identified.

ENERGY ESTIMATION

The mass vector has to be determined in order to estimate the subsystems energies from equation (28). The individual subsystems masses are generally estimated using a power injection method [10] applied to each subsystem:

$$\overline{y_i} = \omega \eta_{tot,i_i} m_i \overline{v_i^2} \tag{34}$$

Where $\overline{y_i}$ is the active injected power in the excited subsystem Ω_i, $\overline{v_i^2}$ is the mean squared velocity over Ω_i and $\eta_{tot,i}$ is the total apparent damping loss factor of the subsystem i.

The so-called equivalent mass of a subsystem is then computed as:

$$m_i = \frac{\overline{y_i}}{\omega_B \eta_{tot,i} \overline{v_i^2}} \quad (35)$$

The total loss factor can be reasonably estimated from reverberation time measurements at various positions within Ω_i. It is classically expressed as:

$$\eta_{tot,i} = \frac{13.8}{\omega_B \overline{T_i}} \quad (36)$$

where $\overline{T_i}$ is the frequency band and space average of the structural reverberation time.

LOSS MATRIX IDENTIFICATION

The balance equation of each of the subsystems when they are independently excited at a turn by a given power -the most complete experience that can be built from the structure- provides $N_\Omega{}^2$ equations, which is enough to identify all loss factors.

When each subsystem is excited at a turn by a unit power, energies in each subsystem are related to the loss matrix by:

$$\omega_B \mathbf{L}\mathbf{E} = \mathbf{I} \quad (37)$$

Where $\mathbf{E}$ the transfer energy matrix of which term $\mathbf{E}_{ij}$ denotes the response of subsystem i when subsystem j is excited.

The loss matrix can then be identified as:

$$\mathbf{L} = \frac{1}{\omega_B} \mathbf{E}^{-1} \quad (38)$$

Nevertheless, a direct inversion of the transfer energy matrix generally leads to many negative loss factors. One reason could be the developed theory is not appropriate to describe the reality. Another reason, dominating until now, is that the inversion is very sensitive to the measuring noise.

Taking advantage of its peculiar matrix structure, N. Lalor [8] has shown that the energy transfer matrix inversion procedure could be more efficient –less noise sensitive– by solving separately the damping loss factors, from the coupling loss factors. The Lalor's technique displays another advantage; it allows the simplification of the SEA network, by considering only the relevant coupling loss factors. This method has been widely used throughout the SEA-XP software [11] to solve many industrial problems. Nevertheless, its practical application remains very sensitive to measurement quality. It is especially the case for heterogeneous structures such as car bodies, where point mobilities can range over 10 dB in a given subsystem.

ENHANCED IDENTIFICATION OF THE VIRTUAL SEA MODEL

VSEA as a numerical technique was build in order to improve ESEA by taking advantage of numerous available data to improve the statistics and full model consistency avoiding measurement noise. Following the developments of the second section, it appears that a more accurate statistical modeling can be performed using equations (26), becoming (32) after inversion. The same procedure as for ESEA is applied to identify the modal loss matrix, except that the procedure previously used to determine the damping loss factor –already known as an FE model parameter- is now used to extract the modal densities.

MODAL DENSITIES DETERMINATION

The equations for modal density identification are obtained by writing the energy conservation in the system (i.e. the sum of dissipated powers into subsystem is equal to the sum of injected powers). Considering the definition of the loss matrix (30) and definition (33), the summation of the lines of equation (33) leads to:

$$\omega_B \overline{\mathbf{e}_\mathbf{n}}^t \mathbf{diag}\{\boldsymbol{\eta}\} \cdot \mathbf{n} = \sum \overline{\boldsymbol{\pi}} \quad (39)$$

where $\boldsymbol{\eta}$ is the vector of the damping loss factors attached to each sub-system and known from the model data.

At the moment, we only consider the case of a uniformly damped structure, that is to say:

$$\mathbf{diag}\{\boldsymbol{\eta}\} = \eta \mathbf{I} \quad (40)$$

When only one subsystem is excited by a unit-power, $\sum \overline{\boldsymbol{\pi}} = 1$ and one obtain the following set of equations:

$$\eta \omega_B \mathbf{E}_\mathbf{n} \cdot \mathbf{n} = \mathbf{1} \quad (41)$$

Where $\mathbf{1}$ is a vector of which elements are equal to 1 and dimension is the number of subsystems

Finally, the modal density appears to be:

$$\mathbf{n}_{B\Omega} = \frac{1}{\eta \omega_B} \mathbf{E}_{\mathbf{n,B,\Omega}}{}^{-1} \cdot \mathbf{1} \quad (42)$$

SEA LOSS MATRIX CALCULATION

Starting with expression (26), one can compute the SEA loss matrix using relations (32), (33) and (42) as :

$$\mathbf{L} = \frac{1}{\omega_B}\mathbf{E}_{\mathbf{n}}^{-1}\mathbf{diag}\left\{\mathbf{n}\right\}^{-1} \tag{43}$$

It is noticeable here, that the SEA system has been identified without reference to the sub-system masses. This comes from the fact that subsystem energy and velocity are both computed from the modal energy. Identifying the modal energy expressions (13) and (33), introducing expression (23) leads to

$$\frac{1}{4}\mathbf{S}_{\Omega}\mathbf{Y}^{-1}\mathbf{S}_{\Omega}^{++}\overline{\mathbf{v}^2} = \mathbf{diag}\left\{\mathbf{n}\right\}^{-1}\cdot\bar{\mathbf{e}} \tag{44}$$

Relation (44) is then identified to expression (29) to get the subsystem equivalent masses:

$$\mathbf{diag}\left\{\mathbf{m}\right\} = \frac{1}{4}\mathbf{diag}\left\{\mathbf{n}\right\}\overline{\mathbf{Y}}^{-1} \tag{45}$$

where $\overline{\mathbf{Y}} = \left(\mathbf{S}_{\Omega}\mathbf{Y}^{-1}\mathbf{S}_{\Omega}^{++}\right)^{-1}$ is the diagonal matrix of the mean mobilities of the subsystems

This expression (45) is proposed by many authors to estimate either $\mathbf{m}$, $\mathbf{n}$ or $\mathbf{Y}$ when the two other variables are fixed. Due to the fact that all matrices are diagonal, expression (45) applies independently for each subsystem.

MEASURE OF THE MODELING IMPROVEMENT

The modification of the formulation that is proposed leads to cumulative positive effects: the modal energy transfer matrix provides a better definition of the optimal sub-structuring and at the same time subsystems internal variance is reduced. The performance of the Virtual SEA model is checked by comparing the difference between the initial and the reconstructed transfer matrix $\left\langle\mathbf{T}^2\right\rangle_B$.

The reconstructed solution is built by substituting point-modal energies in each subsystem, by their mean values, obtained from the VSEA model.

The reconstructed vector of the point modal energies is more mathematically obtained by inverting expression (23) :

$$\hat{\mathbf{e}}_{\mathbf{n}} = \mathbf{S}_{\Omega}^{++t}\cdot\overline{\mathbf{e}_{\mathbf{n}}} \tag{46}$$

Using equations (17), (19), (24), (43), and (46), the point to point transfer matrix can be reconstructed from the SEA loss matrix, as:

$$\hat{\mathbf{T}}^2 = \frac{4}{\omega_B}\mathbf{Y}\mathbf{S}_{\Omega}^{++t}\mathbf{diag}\left\{\mathbf{n}\right\}^{-1}\mathbf{L}^{-1}\mathbf{S}_{\Omega}^{++}\mathbf{Y} \tag{47}$$

It may be noticed here that the

Various metrics can be used to measure the reconstruction error. It was found convenient, for a model built for engineering purposes, to use the difference of transfer levels in dB as an error indicator:

$$\boldsymbol{\varepsilon} = 10Log\left(\frac{\hat{\mathbf{T}}^2}{\mathbf{T}^2}\right) \tag{48}$$

The statistics (mean values, variance or histograms) of the error can then be calculated at the overall system level or at the subsystem level. Figure 7 later shows an example of map of the error within each subsystem for a body-in-white at 630 Hz.

FRF COMPUTATION IMPROVEMENT

FRF SYNTHESIS PRINCIPLE

The proposed technique relies on the computation of large size and band-averaged transfer matrices like the transfer matrix $\left\langle\mathbf{T}^2\right\rangle_B$ of equation (19). From ESEA experience, we know that around 20 nodes are required per subsystem to get a satisfactory low variant estimate of the mean squared transfer velocity and that around 50 subsystems are expected for describing mid-frequency interactions in a car body, the total number of observation nodes should be at least of about 1000 for a full body-in-white FE model. That means the synthesis of 9 million of FRFs as the local transfer is estimated in X, Y, Z directions at each nodal point. Obviously commercial FE post-processors are not designed for such an intensive task and it was decided to develop a dedicated FRF solver for computing the mean squared velocity matrix in decent time.

NASTRAN is used as the core FE solver to extract the eigenvalues of the dynamic problem (1), ω_m^2, and to store the related mode shape amplitudes at the observation nodes, $\boldsymbol{\varphi}_m$. The mode shapes are normalized such as $\boldsymbol{\varphi}_m{}^t\mathbf{M}\boldsymbol{\varphi}_m = 1$.

Assuming a diagonal, frequency-independent, modal damping, such as $\boldsymbol{\varphi}_m{}^t\mathbf{K}^{-1}\mathbf{D}\boldsymbol{\varphi}_m = \eta_m$, the transfer matrix defined by equation (3) can be written as a modal sum:

$$\mathbf{T} = \sum_m h_m(\omega)\mathbf{T}_m \tag{49}$$

where $\mathbf{T}_m = \boldsymbol{\varphi}_m\boldsymbol{\varphi}_m^t$ and $h_m(\omega) = \dfrac{j\omega}{\omega_m^2 - \omega^2 + j\eta_m\omega\omega_m}$ is the generic modal response function.

Using definition (4), the input mobility vector appears to be:

$$\mathbf{y} = \sum_m \mathrm{Re}\left\{h_m(\omega)\right\} diag\left\{\mathbf{T}_m\right\} \quad (50)$$

The band-averaged squared value of the transfer matrix, as used in equation (19), is obtained as:

$$\left\langle T^2 \right\rangle_B = \frac{1}{|B|}\int_B \left| \sum_m h_m(\omega)\ \mathbf{T}_m \right|^2 d\omega \quad (51)$$

In a same manner, the band averaged input mobility is computed as:

$$\left\langle \mathbf{y} \right\rangle_B = \frac{1}{|B|}\int_B \sum_m \mathrm{Re}\left\{h_m(\omega)\right\} diag\left\{\mathbf{T}_m\right\} d\omega \quad (52)$$

Several FRF solvers have been developed to speed up computation and decrease memory consumption, when computing expressions (51) and (52).

FULL MODAL INTEGRATION SOLVER

The first solver is an exact –but slow- solver. The modal integration is always performed on the full set of modes. The generic modal response function is sampled for each mode and stored as a 2D array [frequency, mode]. The tensor product $h_m(\boldsymbol{\omega}, \omega_m)\mathbf{T}_m$ is then vectorized, while reciprocity is used to compute only half of the transfer matrix. After a cumulative sum of modal terms, all narrow band FRFs are band-averaged.

For automotive applications in the mid frequencies (f<900 Hz), the number of modes may be as many as 5000 and the number of points necessary to build a VSEA model overpass 1000. Therefore, the synthesis CPU time and required memory are both quite large. Nevertheless, this option was kept as a reference solution.

BAND OPTIMIZED SOLVER

In each band, the modal synthesis is limited to the modes of which eigenfrequency remains within the band (resonant modes). Using strict selection criteria, the lowest analysis bands may be empty of resonant mode. These bands are thus filled up with the closer off-band modes in order to have at least 5 modes (user-defined value) in any studied frequency band. This simple selection of modes provides a speed-up factor of about 10. The associated error is very low.

BAND OPTIMIZED AND ANALYTICAL INTEGRATION

The frequency integral operator of equation (51) can be moved inside the modal summation symbol and developed in the following way:

$$\left\langle \mathbf{T}^2 \right\rangle_B = \frac{1}{|B|}\left[\sum_m \mathbf{T}_m^2 \int_B \left|h_m(\omega)\right|^2 d\omega + 2\sum_m \sum_{m'} \mathbf{T}_m \mathbf{T}_{m'} \int_B \mathrm{Re}\left\{h_m(\omega) h_m^*(\omega)\right\} d\omega \right] \quad (53)$$

Both frequency integral of equation (53) have analytical expressions obtained by integration in the complex-plane [13], which in a way simplify the computations. Due to the modal cross-terms, on the other hand, the required number of operations is largely increased. As $\int_B \mathrm{Re}\left\{h_m(\omega) h_m^*(\omega)\right\} d\omega$ is rapidly decaying when m and m' correspond to modes with distant eigenfrequencies, the double summation can be also truncated, taking into account only interacting modes. The selection of cross terms is governed by a user-defined parameter, that can be used to balance between solution accuracy and CPU time.

The time to get the FRF matrix can thus be tuned at convenience by controlling the amount of processed information. The 9 millions of required FRF can be generated in less than an hour or over several days on a standard PC computer, depending on acceleration –precision– options.

A similar treatment is performed on the real part of the input mobility, that doesn't need any further simplification:

$$\left\langle \mathbf{y} \right\rangle_B = \frac{1}{|B|}\sum_m diag\left\{\mathbf{T}_m\right\} \int_B \mathrm{Re}\left\{h_m(\omega)\right\} d\omega \quad (54)$$

SUBSTRUCTURING PROCEDURE IMPROVEMENT

To get a pertinent SEA subsystem partition for any FE model, whatever is its degree of structural complexity, an automated process seems the more reasonable solution. This process called sub-structuring is not trivial in a car due to the dynamical "morphing", i.e. the fact that, depending on the frequency, a part of the structure can switch from one behavior (a hollow body) to another (a shell assembly). As a consequence, the resulting sub-structuring is expected to be changed from one frequency band to another. As far as the modal density increases –which is globally the case in a car in the mid–frequencies– the number of subsystems increases. In a car, the number of subsystems should reach an asymptote above a given frequency where the whole structural complexity has been revealed, that is to say when it may be seen as an ensemble of connected plates. For a car body, this frequency is higher than 2 kHz.

As stated above, the sub-structuring has to be pertinent, in the sense that -characterized by a reduced modal energy matrix- it is able to be identified to a SEA model. The performance of a given sub-structuring is related to

the precision of the associated SEA modeling; thus, expression (48) can be used to build various metrics for the sub-structuring performance, as it was shown previously.

From early studies, several algorithms have been either developed or checked to group the nodes in the most SEA-consistent way. The latter involves attractive, genetic, dichotomic and entropic algorithms that could be mixed up or chained [Ref 13]. Among various algorithms, the so-called “attractive method” seems to give the best results and most of the development has been focused on it.

ATTRACTIVE SUB-STRUCTURING METHOD

Let us assume P is an initial partition of the considered structure into subsystems and is made of N groups of nodes, denoted P_n. The point-to-point transfer inside a group is referring to the related nodes k and l. One can define the probability of one particular node k to be part of a subsystem as:

$$\mathrm{Prob}\left\{k \in P_n\right\} = \frac{\left\langle \widehat{\overline{v}}_{kl}^{2} \right\rangle_{l \in P_n}}{\left\langle \sum_{k' \in P} \widehat{\overline{v}}_{k'l'}^{2} \right\rangle_{l' \in P}} \qquad (55)$$

This probability is 1 in case $P_n = P$ (1-subsystem partition). If the average is restricted to excitation nodes l' inside the subsystem defined by P_n, the probability is less than 1. Given a partition and the node k , one can check its probability to be part of an another subsystem P_m as

$$\mathrm{Prob}\left\{k \in P_m\right\} = \frac{\left\langle \widehat{\overline{v}}_{kl}^{2} \right\rangle_{l \in P_m}}{\left\langle \sum_{k' \in \Omega} \widehat{\overline{v}}_{k'l'}^{2} \right\rangle_{l' \in \Omega}}$$

Thus, this probability can be considered as an attraction factor of a node to a subsystem. In each subsystem, there is a node of lowest probability. This node can thus be moved to the partition of maximum attraction. Node-probabilities need then to be re-evaluated and the process can be re-iterated in a loop till it comes to an end. Nevertheless, no convergence criterion was found to this process and the flux of nodes, within each iteration, depends on the initial partition.

INITIAL PARTITION

This initial partition is generated by applying a given threshold to the modal transfer energy matrix, starting from the matrix maximum. All nodes within the threshold are gathered (threshold group, tg) and removed from the matrix. This process is re-iterated until all nodes have been allocated to subsystems. In practice, the partition-initialization routine is more complex as a subpartition (stg) of tg is also performed by applying the threshold to all nodes part of tg. The final retained tg set is obtained by selecting nodes satisfying to the reciprocity principle: two nodes both excited and observed must be included in the final tg set.

The most convenient threshold seems to be the mean of the modal transfer energy matrix. Nevertheless, as mentioned earlier, it was observed some dependence of sub-structuring to the threshold value.

ALGORITHM STABILIZATION

After having improved the computational performance of the attractive method, it was possible to perform series of sub-structuring related to initial partition threshold value.

For each sub-structuring, a correlation matrix is computed between nodes. The correlation matrix $\mathbf{C}$ is such as $C_{kl} = 1$ when nodes k and l belong to the same subsystem, $C_{kl} = 0$ when they don’t. $\mathbf{C}$ matrices are averaged over all sub-structuring series. At the end, nodes with a correlation value equal to 1 are part of the same subsystem over all partitions. These nodes provide a very robust and stable sub-structuring and this sub-structuring is called sub-structuring attraction kernel. After the attraction kernel is identified, remaining nodes are associated to their most probable subsystem using relation (55). This method is very fast as the 1200 nodes of a car body can be grouped into 1 hour into subsystems on a standard PC (see Figures 2, 3, 4).

The sensitivity to node choice and sampling is widely reduced by the new variables introduced in equation (26). This is illustrated in Figure 5 where the transfer functions at dash panel nodes are compared with and without mobility normalization.

The identification of CLF parameters is then performed by Lalor's method and the number of unknown CLF's is user's defined. As default all subsystems are cross-coupled as in Figure 5. The quality of transfer reconstruction can be now checked by comparing initial and reconstructed transfers, following equation 47. Figure 7 shows the mean matrix error in dB (eq. 48) between both transfers when averaged over directly and indirectly excited subsystem domains (left) and the histogram of errors (right). The latter shows that 95 % of transfer functions lie within 2 dB error. The full initial 1217 x 12147 FRF matrix has thus been packed into a small Loss matrix of 34 x 34 able to reconstruct the vibration field within 2dB errors.

Virtual SEA models provide most of the classical SEA features such as power flow analysis as shown on figure 8. As expected the power flow is larger in directly connected structures.

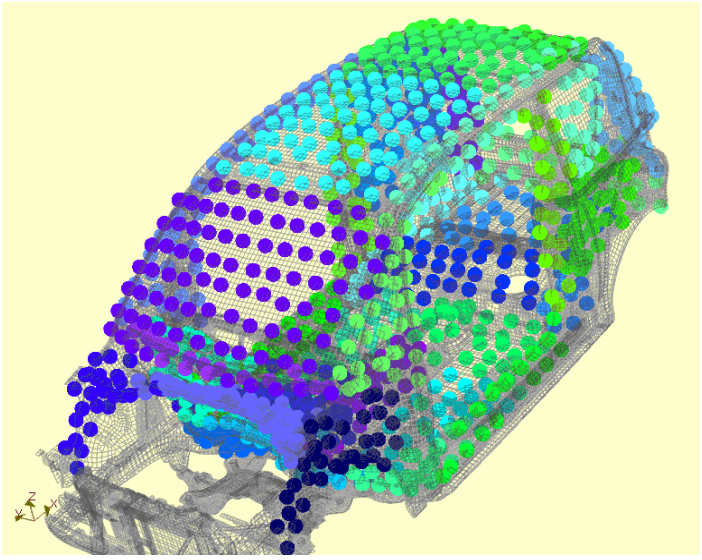


Figure 2: Full substructuring of a body-in-white, using the attractive method. A total of 34 subsystems is found. A subsystem is a collection of nodes, each node being at the same time an excited and a response node used to create the FRF energy matrix

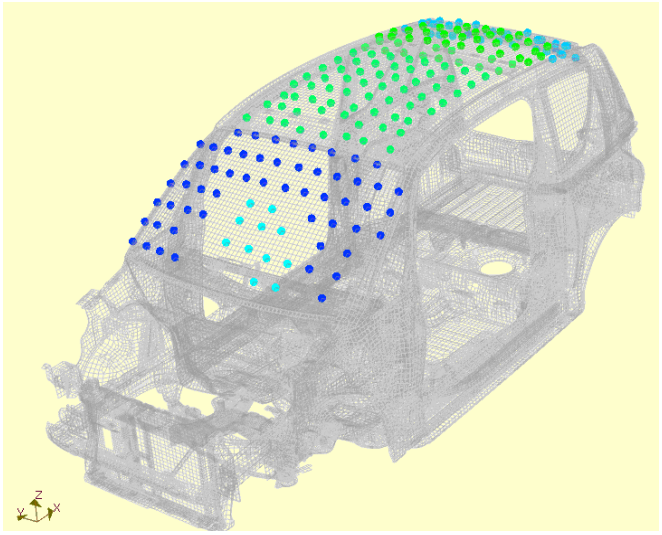


Figure 4: Substructuration details, the attraction kernel; Nodes are grouped together by the attractive algorithm. The more robust grouping is the kernel of the attraction method. The kernel includes only nodes always found together in the various sets obtained for different thresholds of the FRF matrix. Compared to subsystems in Fig. 3, the wind screen is here made of two groups of the kernel nodes. The roof is split into four kernel groups. After attraction of remaining nodes on the kernel, the final substructuration is obtained as shown in Fig. 2 and 3.

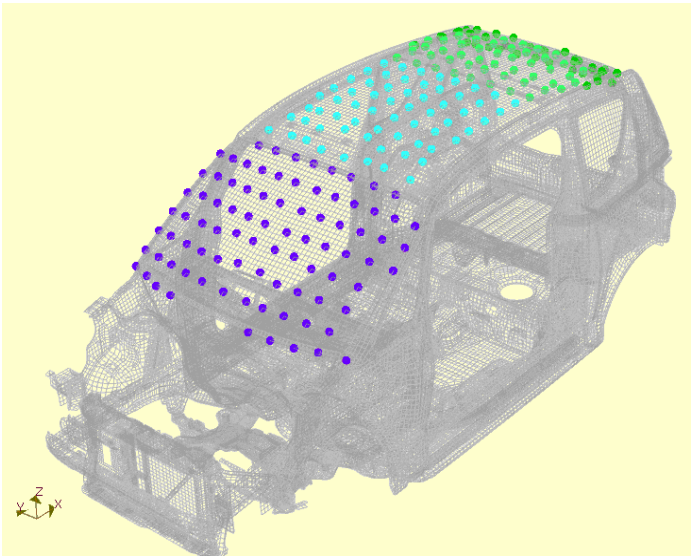


Figure 3: Car-body auto-substructuring details; windscreen and roof nodal partition. The windscreen is seen as 1 full subsystem while the roof is subdivided into 3 subsystems.

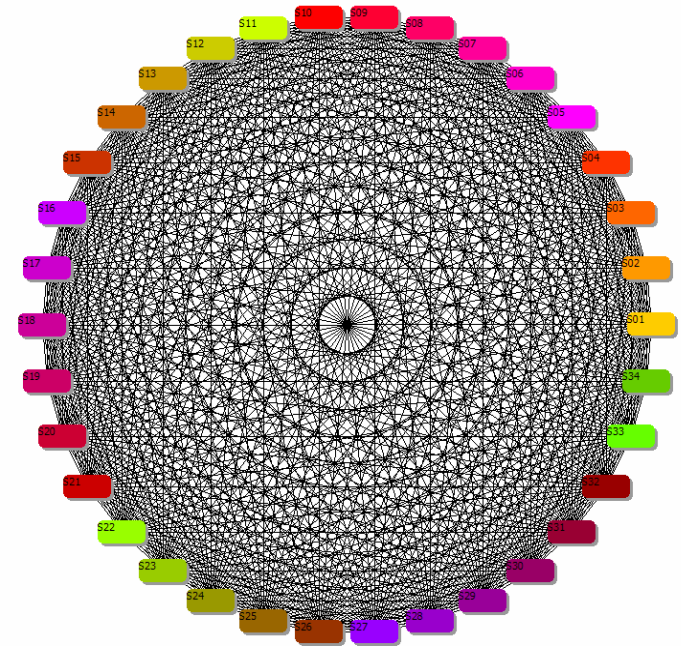
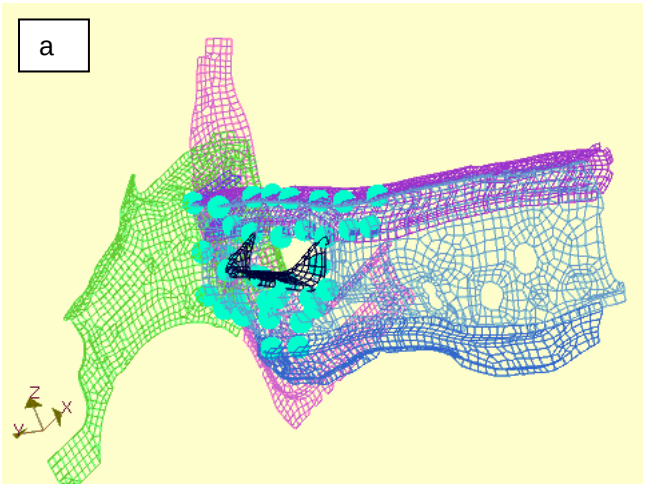


Figure 5: View of Fig 2 model in the SEA inverse solver. Each box is a subsystem and in-between are drawn the junctions. To identify the CLF, all subsystems are crossed coupled together per default and there is no need to a priori detect the physical connections between subsystems. After CLF are obtained, it is possible to automatically remove weak links (i.e. junctions that always carry very low power in all) possible excitation configuration) in order to simplify the model and limit the number of parameters



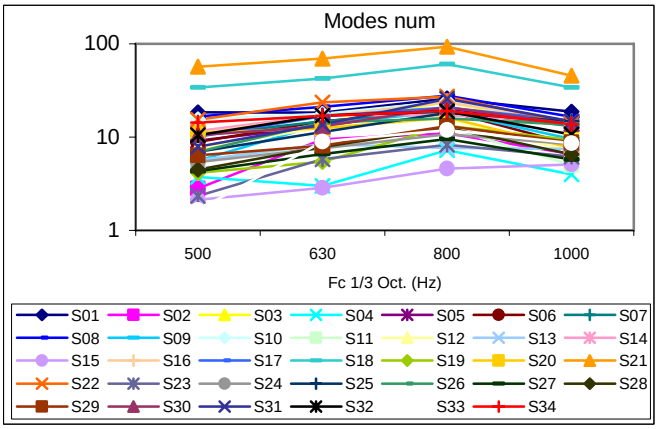


Figure 6: Number of modes per 1/3rd octave band as estimated from the 34 SEA-body-in-white model of figure 4

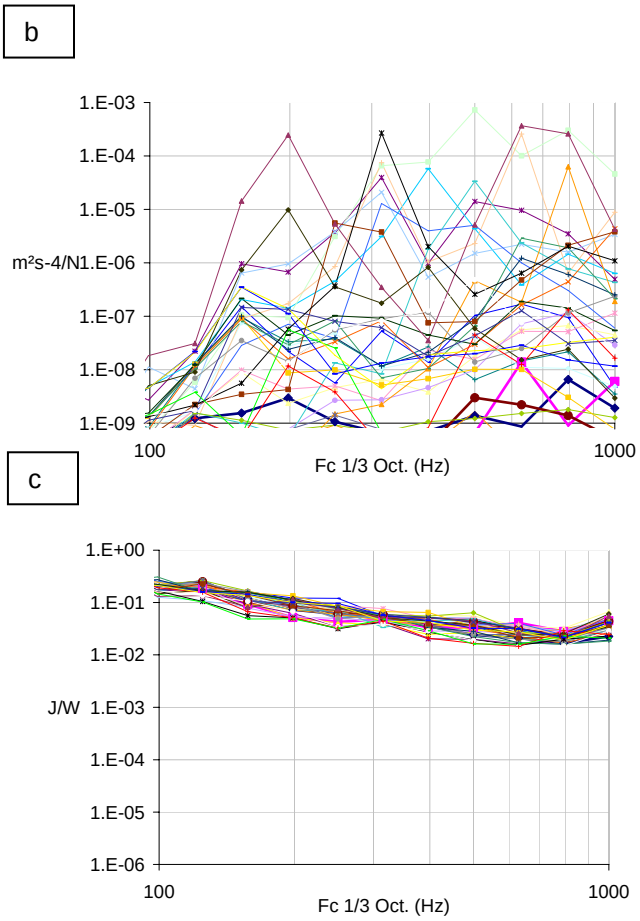


Figure 5: Example of scattering of transfer mobility in the dash panel (a) Nodes of subsystem 9 (part of dash panel) where the transfer mobility is computed;
(b) The various transfer mobility T99 (within subsystem 9 self-excited by unit-force at all internal nodes locations);
(c) Same transfers but in modal energy transfers En99 (within subsystem 9 self-excited by unit-power). By going to local velocity² to modal energy using appropriate normalization the scattering is reduced to less than 3 dB, meaning the modal energy matrix becomes insensitive to node sampling

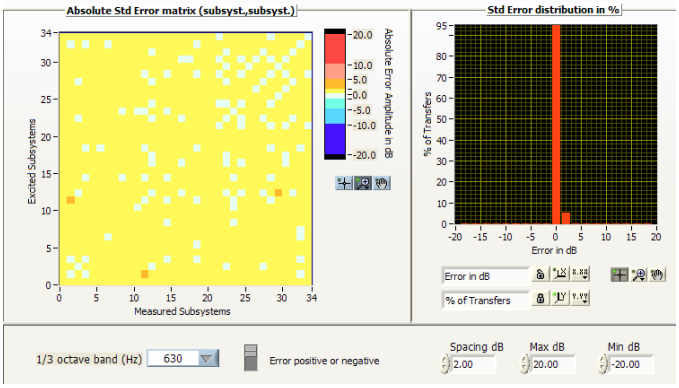


Figure 7: On the right, matrix error between the initial modal transfer energy matrix and the reconstructed modal transfer energy matrix. Color scale is in dB. On the left, percentage of reconstructed transfer within a given error interval in dB

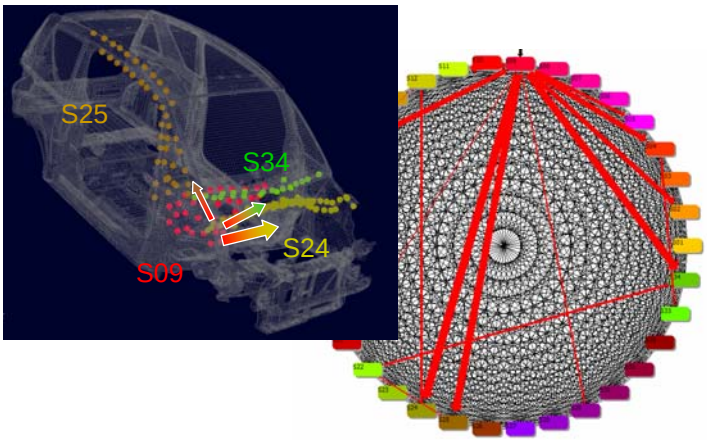


Figure 8: Main power flows within 10 dB range in 630 Hz band when exciting the dash panel S09

CONCLUSION

In this paper, the previously published Virtual SEA method was first modified in order to obtain a more accurate prediction of point-to-point structural transfer functions, using a statistical treatment of FE results, later identified to a SEA model. A SEA model is useful in a design process because it allows investigating the effects of local structural modification (including damping) on the structure response to any vibration sources. Coupling loss factors are nearly independent of damping at least in the frequency range where weak-coupling assumption is verified between subsystems. It is why coupling loss factors provided by virtual SEA under uniform damping assumption are robust as far as subsystems remain weakly coupled. Measured damping loss factors can thus be provided separately to the virtual model with good predictive results as shown in [3].

The applicability to industrial problems -such as a car body modeling in the mid-frequencies- has been greatly improved by working out the numerical process of the two most time consuming tasks: the FRF synthesis and the automatic sub-structuring process. The whole virtual SEA process can now be performed within a few hours for an average size (150000 nodes) car body FE model, in the range 200-800 Hz. Of course, Virtual SEA predictions quality will be strongly related to the FE model construction, which is another topic.

There are many perspectives to this work. Let us just mention two of them:

- The Virtual SEA model has to be properly connected to cavities. The space-frequency information required for sound radiation computation will have to be generated from the copious data now available mainly by estimating the mean subsystem wave number.

- Confidence level of the predictions has to be determined when accounting for the various uncertainties occurring throughout an industrial process. Recent work on stochastic modeling [Ref 14] allows the determination of FRF's confidence interval due to any model change within a given range, from which the SEA statistics can be extended to industrial products ensemble.

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REFERENCES

[1] Borello G. "*Identification of SEA Coupling Loss Factors on a liquid rocket engine*", Inter-Noise 1991, Sydney, Australia
[2] Borello G. "*Prediction and control of structure-borne noise transfers in vehicules using SEA*", Euro-noise, 4-7 October, 1998, Munich, Germany
[3] Gagliardini L., Houillon L., Petrinelli L., Borello G. "*Virtual SEA: mid-frequency structure-borne noise modeling based on Finite Element Analysis"*, SAE-NVC 2003-01-1555, Traverse City, MI, USA
[4] Ohayon, R., Soize C. "*Structural Acoustics and Vibration*", 1998, Academic Press
[5] Bathe K. J. and Wilson E.L. "*Numerical Methods in Finite Element Analysis*", Prentice-Hall, 1976, New York
[6] Maidanik G., "*Response of coupled dynamic systems"*, Journal Sound Vib., 1976, vol 46, pp 561-583
[7] Mondot J.-M., Petersson B. "*Characterization of structure-borne sound sources, the source descriptor and the coupling function"*, Journal Sound Vib., 1987, 114-3, pp 507-518
[8] Lalor N. *"The Experimental Determination of Vibrational Energy Balance in Complex Structures",* Paper 108429 Proc. SIRA Conference on Stress & Vibration, 1989, London
[9] Rosen M., Borello G. *"Damping and coupling loss factors estimation in SEA method: what is really measured?"*, Inter-Noise 1996
[10] Lalor N. "*Experimental statistical energy analysis: a tool for the reduction of machinery noise"*, Presented to the E.J. Richards Memorial Session at the 131st ASA Meeting, Indianapolis, USA, May 1996
[11] SEA-XP User Guide Manual, InterAC Sarl, Version 2.6, October 2006
[12] Arfken G. *"Cauchy's Integral Formula",* §6.4 in Mathematical Methods for Physicists, 3rd ed. Orlando, FL: Academic Press, 1985
[13] Crosby Jack L., "*Computer Simulation in Genetics",* John Wiley & Sons, 1973, London
[14] Durand J.-F., Gagliardini L., Soize C. "*Nonparametric modeling of the variability of vehicle vibroacoustic behavior*", SAE-NVC 2005-01-2385, Traverse City, MI, USA
[15] Manning Jerome E. "*SEA Models to Predict Structureborne Noise in Vehicles*" SAE-NVC 2003-01-1542, Traverse City, MI, USA

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